Extended Planck Scale

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Abstract

Traditional derivations of the Planck mass ignore the role of charge and spin in general relativity. From the Kerr-Newman null surface and horizon radii, quantized charge and spin dependence are introduced in an extended Planck scale of mass. Spectra emerge with selection rules dependent upon the choice of Kerr-Newman radius to link with the Compton wavelength. The appearance of the fine structure constant suggests the possibility of a variation in time of the extended Planck mass, which may be much larger than the variation in the traditional one. There is a suggestion of a connection with the α value governing high-energy radiation in Z-boson production and decay.

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Different arguments have led researchers to a measure of the scale at which gravity must necessarily mesh with quantum theory, the *Planck scale*. The most common approach is to form a combination of the gravitational constant G, the reduced Planck constant $\hbar$, and the speed of light c that has the dimension of mass and label it the *Planck mass* m_p

$$m_p = \sqrt{\frac{\hbar c}{G}} \simeq 2.2 \cdot 10^{-5}\,\mathrm{g} \tag{1}$$

or, equivalently, one can consider the Planck length $l_p = \sqrt{G\hbar/c^3} \simeq 1.6 \cdot 10^{-33}$ cm, the Planck time $t_p = l_p/c \simeq 5.4 \cdot 10^{-44}$ s, or the Planck energy $E_p = m_p c^2 \simeq 1.3 \cdot 10^{19}$ GeV.

However, this approach does not distinguish between Newtonian gravity and general relativity, the preferred *relativistic* theory of gravity. A more illuminating argument reflecting both the quantum scale and the role of general relativity derives from equating the Compton wavelength of a particle of mass m, namely $\lambda_C = \hbar/mc$, with its gravitational radius $r_S = 2Gm/c^2$, the radius of its event horizon as found from the Schwarzschild metric. This gives the same result apart from a factor $1/\sqrt{2}$. In following this procedure, what is made transparent is that the "Planck particle" so derived is without spin or charge. However, spin and charge are the fundamental quantized aspects of matter. To exclude them is to ignore the important couplings that spin and electromagnetism have to gravitation. Therefore to be general, we consider what effect their consideration has on what we now designate as the *extended Planck* (henceforth referred to as *"plex"*) scale. In place of the spinless neutral Schwarzschild particle, we consider a particle endowed with spin and charge, again within the context of general relativity. The metric for a body of mass m, charge q and angular momentum per unit mass a is the Kerr-Newman metric (with $c = G = 1$) [1]

$$ds^2 = \frac{sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right) d\phi - a\, dt\right]^2 - \frac{D}{\rho^2}\left[dt - a\sin^2\theta\, d\phi\right]^2 + \frac{\rho^2}{D}\, dr^2 + \rho^2 d\theta^2\ , \tag{2}$$

where

$$D \equiv r^2 - 2mr + a^2 + q^2 \;, \qquad \rho^2 \equiv r^2 + a^2 \cos^2\theta \;. \tag{3}$$

The new gravitational radius is (we now restore the c and G) [1]

$$r_+ = \frac{G}{c^2}\left(m + \sqrt{m^2 - \frac{q^2}{G} - \frac{c^2}{G^2}\,a^2}\right) \;. \tag{4}$$

However, with spin and charge added, there is scope to focus on the other significant radius, the radius of the"null surface" r_- with the negative sign in front of the square root

$$r_- = \frac{G}{c^2}\left(m - \sqrt{m^2 - \frac{q^2}{G} - \frac{c^2}{G^2}\,a^2}\right) \;. \tag{5}$$

Since we are dealing with the quantum domain, we quantize the charge in units of the charge e of the electron and the angular momentum in units of the fundamental quantum of angular momentum $\hbar$, with respective quantum numbers N and s:

$$q = N\,e \;, \qquad a = s\,\frac{\hbar}{m} \;. \tag{6}$$

(Note that the m appears again through the spin.) When one sets the Kerr-Newman event horizon (eq. (4)) and null surface (eq. (5)) radii of the particles equal to their Compton wavelengths, and substitutes the quantized charge and spin from eq. (6), one has

$$\frac{\hbar}{mc} = \frac{G}{c^2}\left(m \pm \sqrt{m^2 - \frac{N^2 e^2}{G} - \frac{c^2 \hbar^2\, s^2}{G^2 m^2}}\right) \;. \tag{7}$$

At this point in the nascent state of development of the subject, it is unclear whether it is r_+ or r_- that should be the length scale to connect with the Compton wavelength in the quantum domain or indeed, if both values have a role to play. Accordingly, in what follows, both possibilities will be investigated. Solving for m, one finds that the mass which we now refer to as the *extended Planck mass* m_{plex} is

$$m_{plex} = m_{pl} \sqrt{\frac{2(1+s^2)}{2-\alpha N^2}} \, , \tag{8}$$

for both cases, where $\alpha \equiv e^2/\hbar c \simeq 1/137$ is the fine structure constant and henceforth we use the subscript notation "pl" to designate the standard Planck mass with the $1/\sqrt{2}$ factor included, $m_{pl} \equiv \sqrt{\hbar c/(2G)} = m_p/\sqrt{2}$. By eq. (8), the presence of either spin or charge leads to an increase in the value of m_{plex} as compared to the traditional m_{pl}. Moreover, the presence of the fine structure constant in eq. (8) provides an additional source of interest, given the current focus upon its apparent slow variation in time [2]-[5].

Following recent claims [2]-[5] that the value of the fine structure constant underwent changes during the last half of the history of the universe, we focus on the possibility that α could have had a considerably different value in the still more distant past. Although rather unorthodox in the low-energy regime, this idea appears quite naturally in the context of renormalization, in which the coupling "constants" are actually running couplings. In the standard model, the early universe expands and cools precipitously in its very first instants when it emerges from the big bang, and the energy scale drops substantially, allowing for significant variations in the values of the running couplings.

It has been claimed that if the fine structure "constant" changes at all, a change in c rather than e is responsible as a change in e would violate the laws of black hole thermodynamics [6]. A time-varying α can be accomodated in the context of varying speed of light cosmologies, of which many proposals have appeared recently [7]-[13] (see however the criticism in Ref. [14]). While the reported variation of α over the last 10^{10}

years is minute (of the order of 10^{-5} [2]-[5]) and the variation of fundamental constants is restricted by primordial nucleosynthesis, it is quite conceivable that more radical changes could have occurred earlier in the history of the universe. Although the current evidence points to a small increase in α as we go forward in time over the time scale thus far surveyed, the essential point is that there is variation and this variation could have been one of decrease from a larger value at a still earlier time. To fix our ideas, suppose that $N = 5$ and s is of order unity. Then, if at sometime in the past, α assumed a value close to $8 \cdot 10^{-2}$ (approximately one order of magnitude larger than its present value), the value of the extended Planck mass m_{plex} would have been many orders of magnitude larger than its present-day value, regardless of the value of the quantum number s (larger values of N lead to large effects for smaller variations of α). By contrast, if this change in the value of α was due to the time variation of c, the change in the traditional Planck mass m_{pl} instead would be relatively insignificant. Since there has been some debate as to whether it is a variation in e or in c that has been responsible for the observed change in α, we point out that the effects in the two cases upon the value of m_{plex} are different. If it is e that varies, this appears only in α in m_{plex} whereas if it is c that is responsible, this change affects another part of the m_{plex} expression as well.

Extremal values are generally useful to gain insight and hence it is perhaps worth noting that the critical upper-limit N value in eq. (8) is $N = 16$ for the present α value of 1/137.036. With this N value, the extended Planck scale becomes infinite for an α value of 1/128. Interestingly, the α value governing high-energy radiation in Z-boson production and decay has been measured to be 1/127.934, suggesting that there really may be some connection between fundamental constants and integers (recalling the history of theorizing about the number 137).

It is to be noted that the scope for the extension of the Planck scale is severely limited if one were to be restricted by the choice of the event horizon radius eq. (4) as opposed to the null surface radius eq. (5). From eq. (7) with the positive sign in front of the square root, one finds the inequality

$$\frac{\hbar}{mc} - \frac{Gm}{c^2} \geq 0 \tag{9}$$

and hence, with eq. (8)

$$m_{pl} \leq m_{plex} \leq \sqrt{2}\, m_{pl} \tag{10}$$

These conditions in conjunction with eq. (8) place the following restrictions on the allowed spin and charge quanta:

$$s^2 + N^2\alpha \leq 1\ , \qquad N^2\alpha < 2, \tag{11}$$

Thus, the allowed values of s and N for $\alpha = 1/137$ are

a) for $s = 0$, $N \leq 11$

b) for $s = 1/2$, $N \leq 10$

c) for $s = 1$, $N = 0$. Note that spin two is not allowed in this case and this might evoke some surprise as the graviton is seen as a spin two boson. However the extended Planck mass, as the traditional Planck mass, is very large whereas the graviton mass is zero to a very high level of accuracy ($m_{graviton} < 10^{-59}$ g). They are very different concepts.

Given the new extended approach, it is natural to introduce an extended *Planck charge* and a *Planck spin*. These quantities could be defined by assuming that the

"Planck particle" considered is an extremal black hole, i.e. one defined by

$$m^2 = \frac{q^2}{G} + \frac{c^2}{G^2}\, a^2 \tag{12}$$

(corresponding to the equality in (11)) that is maximally charged ($s = 0$, $q = q_{max}$) or maximally rotating ($q = 0$, $s = s_{max}$). These requirements yield the extended Planck quantities

$$q_{plex} = \frac{e}{\sqrt{\alpha}} \simeq 11.7\, e\ , \qquad s_{plex} = 1 \tag{13}$$

(corresponding to the Planck angular momentum $L_{plex} = \hbar$ and now allowing for non-integral N). While q_{plex} is large but not extraordinarily so, L_{plex} is rather ordinary on the scale of particles familiar at an energy much lower than the Planck scale. This is the reason why the inclusion of charge and spin does not appreciably change the value of the extended Planck mass m_{plex} with respect to m_{pl} of the spinless, neutral case, if one assumes that α does not vary. (See, however, below where the null surface radius is used to relate to the Compton wavelength.)

According to the third law of black hole thermodynamics, an extremal black hole corresponds to zero absolute temperature, and is an unattainable state. If the third law survives in the Planck regime, the values of N and s are even further restricted, and the first of (11) should read as a strict inequality.

If one considers instead the null surface of radius r_- defined by eqs. (5) and (8) the inequalities

$$s^2 + N^2\alpha \geq 1\ , \qquad N^2\alpha < 2 \tag{14}$$

follow.

In this case, the allowed values of s and N for $\alpha = 1/137$ are,

a) for $s = 0$, $12 \leq N \leq 16$

b) for $s = 1/2$, $11 \leq N \leq 16$

c) for $s = 1$, $0 \leq N \leq 16$

d) for $s = 2$, $0 \leq N \leq 16$

In this case, spin two is readily allowed.

Particle masses get renormalized and hence behave like running couplings. Perhaps this is the case as well for the extended Planck mass although this is speculation in the absence of a renormalizable theory of quantum gravity. Perhaps what we have in the substance of the extended Planck mass is a semi- classical analogue of renormalization.

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